

RELATIVIZATION IN N-ARY TOPOLOGY

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ABSTRACT

Nithyanantha Jothi and Thangavelu introduced the concept of binary topology in 2011. Recently the authors extended the notion of binary topology to n-ary topology where $n > 1$ an integer and studied n-ary closed sets. The purpose of this paper is to study the n-ary subspaces of a n-ary topological space.

KEYWORDS: Binary Topology, n-ary Topology, n-ary Open, n-ary Closed & n-ary Subspace

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1. INTRODUCTION

Nithyanantha Jothi and Thangavelu [5-10] introduced the concept of a binary topology and studied the corresponding closure and interior operators in binary topological spaces. Following this topologists studied the notion of binary topology in soft topological, generalized topological and supra topological settings [1-4]. The authors [11] discussed nearly open sets in binary topological spaces and also they extended binary topology to n-ary topology [12] and discussed n-ary closed sets [13]

2. PRELIMINARIES

Let $X_1, X_2, X_3, \dots, X_n$ be the non empty sets. Then $P(X_1) \times P(X_2) \times \dots \times P(X_n)$ is the Cartesian product of the power sets $P(X_1), P(X_2), \dots, P(X_n)$. Examples can be constructed to show that the two notions 'product of power sets' and 'power set of the products' are different.

Any typical element in $P(X_1) \times P(X_2) \times \dots \times P(X_n)$ is of the form $(A_1, A_2, \dots, A_n)$ where $A_i \subseteq X_i$ for $i \in \{1, 2, 3, \dots, n\}$. Suppose $(A_1, A_2, \dots, A_n)$ and $(B_1, B_2, \dots, B_n)$ are any two members in $P(X_1) \times P(X_2) \times \dots \times P(X_n)$. Throughout this paper we use the following notations and terminologies.

$(X_1, X_2, \dots, X_n)$ is an n-ary absolute set and $(\emptyset, \emptyset, \emptyset, \dots, \emptyset)$ is an n-ary null set or void set or empty set in $P(X_1) \times P(X_2) \times \dots \times P(X_n)$.

$(A_1, A_2, \dots, A_n) \subseteq (B_1, B_2, \dots, B_n)$ if $A_i \subseteq B_i$ for every $i \in \{1, 2, 3, \dots, n\}$ and

$(A_1, A_2, \dots, A_n) \neq (B_1, B_2, \dots, B_n)$ if $A_i \neq B_i$ for some $i \in \{1, 2, 3, \dots, n\}$.

Equivalently $(A_1, A_2, \dots, A_n) = (B_1, B_2, \dots, B_n)$ if $A_i = B_i$ for every $i \in \{1, 2, 3, \dots, n\}$. If $A_i \neq B_i$ for each $i \in \{1, 2, 3, \dots, n\}$ then we say that $(A_1, A_2, \dots, A_n)$ is absolutely not equal to $(B_1, B_2, \dots, B_n)$ which is denoted as $(A_1, A_2, \dots, A_n) \neq (B_1, B_2, \dots, B_n)$. Let $x_i \in X_i$ and $A_i \subseteq X_i$ for every $i \in \{1, 2, 3, \dots, n\}$. Then $(x_1, x_2, \dots, x_n) \in (A_1,$

$A_2, \dots, A_n$ if $x_i \in A_i$ for every $i \in \{1, 2, 3, \dots, n\}$.

Definition 2.1

Let X_i be an infinite set for every $i \in \{1, 2, 3, \dots, n\}$. $(A_1, A_2, \dots, A_n)$ is finite if A_i is finite for every $i \in \{1, 2, 3, \dots, n\}$ and is infinite if A_i is infinite for some $i \in \{1, 2, 3, \dots, n\}$.

Definition 2.2

Let X_i be an uncountable set for every $i \in \{1, 2, 3, \dots, n\}$. $(A_1, A_2, \dots, A_n)$ is countable if A_i is countable for every $i \in \{1, 2, 3, \dots, n\}$ and is uncountable if A_i is uncountable for some $i \in \{1, 2, 3, \dots, n\}$.

Proposition 2.3

$(x_1, x_2, \dots, x_n) \in (A_1, A_2, \dots, A_n)$ iff $(x_1, x_2, \dots, x_n) \in A_1 \times A_2 \times \dots \times A_n$.

The notions of n -ary union, n -ary intersection, n -ary complement and n -ary difference of n -ary sets are defined component wise. Two n -ary sets are said to be n -ary disjoint if the sets in the corresponding positions are disjoint.

Let $X_1, X_2, X_3, \dots, X_n$ be the non empty sets. Let $T \subseteq P(X_1) \times P(X_2) \times \dots \times P(X_n)$.

Definition 2.4

T is an n -ary topology on $(X_1, X_2, X_3, \dots, X_n)$ if the following axioms are satisfied.

- $(\emptyset, \emptyset, \emptyset, \dots, \emptyset) \in T$
- $(X_1, X_2, X_3, \dots, X_n) \in T$
- If $(A_1, A_2, \dots, A_n), (B_1, B_2, \dots, B_n) \in T$ then
 $(A_1, A_2, \dots, A_n) \cap (B_1, B_2, \dots, B_n) \in T$
- If $(A_{1\alpha}, A_{2\alpha}, \dots, A_{n\alpha}) \in T$ for each $\alpha \in \Omega$ then $\bigcup_{\alpha \in \Omega} (A_{1\alpha}, A_{2\alpha}, \dots, A_{n\alpha}) \in T$.

If T is an n -ary topology then the pair (X, T) is called an n -ary topological space where $X = (X_1, X_2, X_3, \dots, X_n)$. The element $x = (x_1, x_2, \dots, x_n) \in X$ is called an n -ary point of (X, T) and the members $A = (A_1, A_2, \dots, A_n)$ of $P(X_1) \times P(X_2) \times \dots \times P(X_n)$ are called the n -ary sets of (X, T) . The members of T are called the n -ary open sets in (X, T) .

It is noteworthy to see that product topology on $X_1 \times X_2 \times \dots \times X_n$ and n -ary topology on $(X_1, X_2, X_3, \dots, X_n)$ are independent concepts as any open set in product topology is a subset of $X_1 \times X_2 \times X_3 \times \dots \times X_n$ and an open set in an n -ary topology is a member of $P(X_1) \times P(X_2) \times \dots \times P(X_n)$.

Lemma 2.5

Let T be an n -ary topology on $(X_1, X_2, \dots, X_n)$. $T_1 = \{A_1: (A_1, A_2, \dots, A_n) \in T\}$ and $T_2, T_3, \dots, T_n$ can be similarly defined. Then $T_1, T_2, \dots, T_n$ are topologies on $X_1, X_2, \dots, X_n$ respectively.

Lemma 2.6

Let $\tau_1, \tau_2, \dots, \tau_n$ be the topologies on $X_1, X_2, \dots, X_n$ respectively.

Let $T = \tau_1 \times \tau_2 \times \dots \times \tau_n = \{(A_1, A_2, \dots, A_n) : A_i \in \tau_i\}$. Then T is an n -ary topology on $X = (X_1, X_2, \dots, X_n)$. Moreover $T_i = \tau_i$ for every $i \in \{1, 2, 3, \dots, n\}$.

Definition 2.7

Let $(A_1, A_2, \dots, A_n)$ be an element of $P(X_1) \times P(X_2) \times \dots \times P(X_n)$. Then $(A_1, A_2, \dots, A_n)$ is n -ary closed in $(X_1, X_2, X_3, \dots, X_n; T)$ if $(A_1, A_2, \dots, A_n)^c = (X \setminus A_1, X \setminus A_2, \dots, X \setminus A_n)$ is n -ary open in (X, T) .

3. THE N-ARY BASIS AND N-ARY SUB BASIS

Let $X_1, X_2, X_3, \dots, X_n$ be the non empty sets. Let $T \subseteq P(X_1) \times P(X_2) \times \dots \times P(X_n)$ be an n -ary topology on $(X_1, X_2, X_3, \dots, X_n)$.

Definition 3.1

A sub collection B of T is called an n -ary basis for (X, T) if every member of T is an n -ary union of members of B and a sub collection S of T is a sub basis for (X, T) if the n -ary intersections of finitely many members of S form an n -ary basis for (X, T) .

Proposition 3.2

A collection B of n -ary elements of $P(X_1) \times P(X_2) \times \dots \times P(X_n)$ is an n -ary basis for some n -ary topology

$T(B)$ on X if and only if

- the n -ary union of all members of B is $(X_1, X_2, \dots, X_n)$ and
- $T(B) = \{G \in P(X_1) \times P(X_2) \times \dots \times P(X_n) : \text{for every } x \in G \text{ there exists a } B \in B \text{ with } x \in B \subseteq G\}$.

Remark 3.3

$T(B)$ is called the n -ary topology generated by B as an n -ary basis.

Proposition 3.4

A collection S of n -ary elements of $P(X_1) \times P(X_2) \times \dots \times P(X_n)$ is an n -ary

Sub basis for some n -ary topology $T(S)$ on X iff

- the n -ary union of all members of B is $(X_1, X_2, \dots, X_n)$ and
- $T(S) = \{G \in P(X_1) \times P(X_2) \times \dots \times P(X_n) : \text{for every } x \in G \text{ there exists } B \text{ and } C \text{ in } S \text{ with } x \in B \cap C \subseteq G\}$.

Remark 3.5

$T(S)$ is called the n -ary topology generated by S as an n -ary sub basis.

Remark 3.6

An n -ary basis or n -ary sub basis of an n -ary topology T completely determines the n -ary topology T .

Proposition 3.7

A collection B of n -ary elements of $P(X_1) \times P(X_2) \times \dots \times P(X_n)$ is an n -ary basis for the n -ary topology $T(B)$ on X

iff

- for every $x \in X$ there exists $B \in \mathcal{B}$ with $x \in B \subseteq X$ and
- for every $x \in X$ and for any two members C and D of $\mathcal{B}$ with $x \in C \cap D$ there exists $B \in \mathcal{B}$ with $x \in B \subseteq C \cap D$.

Remark 3.8

If $\mathcal{B}$ satisfies (i) and (ii) of Proposition 3.7. then $T(\mathcal{B})$ consists of $(\emptyset, \emptyset, \emptyset, \dots, \emptyset)$, $(X_1, X_2, X_3, \dots, X_n)$ and all n -ary unions of members of $\mathcal{B}$.

Proposition 3.9

Suppose $\mathcal{B}$ and $\mathcal{B}'$ generate the n -ary topologies $T(\mathcal{B})$ and $T(\mathcal{B}')$ Respectively on X as n -ary bases. Then $T(\mathcal{B}')$ is finer than $T(\mathcal{B})$ iff for every $x \in X$ and for every $B \in \mathcal{B}$ there exists $B' \in \mathcal{B}'$ with $x \in B' \subseteq B$.

Proposition 3.10

Suppose $\mathcal{B}$ and $\mathcal{B}'$ generate the n -ary topologies $T(\mathcal{B})$ and $T(\mathcal{B}')$ Respectively on X as n -ary bases. Then $T(\mathcal{B}') = T(\mathcal{B})$ iff

- for every $x \in X$ and for every $B \in \mathcal{B}$ there exists $B' \in \mathcal{B}'$ with $x \in B' \subseteq B$
- for every $x \in X$ and for every $B' \in \mathcal{B}'$ there exists $B \in \mathcal{B}$ with $x \in B \subseteq B'$.

4. N-ARY SUBSPACE

Let $X = (X_1, X_2, \dots, X_n)$ and $Y = (Y_1, Y_2, \dots, Y_n)$ such that $Y \subseteq X$.

Definition 4.1

Let T be an n -ary topology on X . Then $T_Y = \{Y \cap A : A = (A_1, A_2, \dots, A_n) \in T\}$.

Proposition 4.2

T_Y is an n -ary topology on Y .

Proof

If $A = (A_1, A_2, \dots, A_n)$ then $Y \cap A = (Y_1, Y_2, \dots, Y_n) \cap (A_1, A_2, \dots, A_n) = (Y_1 \cap A_1, Y_2 \cap A_2, \dots, Y_n \cap A_n)$ is an n -ary element of $P(Y_1) \times P(Y_2) \times \dots \times P(Y_n)$. which in turn is an n -ary element of $P(X_1) \times P(X_2) \times \dots \times P(X_n)$. Therefore it is easy to show that T_Y is an n -ary topology on Y .

Definition 4.3

The n -ary topology T_Y on Y is inherited from T or induced by T . The pair (Y, T_Y) is called the n -ary subspace of (X, T) .

The next six propositions can be easily proved.

Proposition 4.4

If $\mathcal{B}$ is an n -ary basis for T then $\{Y \cap B : B \in \mathcal{B}\}$ is an n -ary basis for T_Y .

Proposition 4.5

If S is an n -ary sub basis for T , then $\{Y \cap B : B \in S\}$ is an n -ary sub basis for $T Y$

Proposition 4.6

Let $A \subseteq Y \subseteq X$. Then A is n -ary closed in $(Y, T Y)$ iff $A = Y \cap F$ for some n -ary closed set F in (X, T) .

Proposition 4.7

Let $A \subseteq Y \subseteq X$. Then

- $(n-ClA)_Y = Y \cap (n-ClA)$
- $Y \cap (n-IntA) \subseteq n-IntA$.

Proposition 4.8

Let $A \subseteq Y \subseteq X$. Then

- If Y is n -ary closed in (X, T) then A is n -ary closed in $(Y, T Y) \Leftrightarrow A$ is n -ary closed in (X, T) .
- If Y is n -ary open in (X, T) then A is n -ary open in $(Y, T Y) \Leftrightarrow A$ is n -ary open in (X, T) .

Proposition 4.9

Let $Z \subseteq Y \subseteq X$. Then $(T Y)Z = T Z$.

CONCLUSIONS

The notions of n -ary basis and n -ary sub basis are discussed in n -ary topological spaces. Further the n -ary subspaces of an n -ary space are characterized by using these notions.

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